Back around 1970, when I was a junior in college, I took an introductory course in point-set topology using this book, which was originally published in 1963 by McGraw Hill. The course was such a success, and the book so popular, that the class requested the professor to continue on next semester to finish the book up. The professor arranged for a “topics” course to run, and we finished off most of the rest of the text. I distinctly remember thinking that this book was perhaps the best mathematics textbook ever written.

Of course, that was the opinion of a 20-year old, who (like most people that age) thought he knew a lot more than he actually did. With the benefit of about 45 years of additional experience, I now realize that this book isn’t quite as perfect as I thought it was; it has some definite flaws that I’ll discuss below. But it was, and still is, a very good book, and it holds up remarkably well; so well, in fact, that I was delighted to see that it is still in print (with a different publisher), and have selected it as the text for a topology course that I am currently scheduled to teach next fall. (I should point out, though, that some weeks after I made that adoption decision, I made the acquaintance of Croom’s excellent book *Principles of Topology*; had I known of this book earlier, I likely would have selected it instead.)

The book under review is divided into three parts (entitled, respectively, “Topology”, “Operators” and “Algebras of Operators”), but I think of it as being divided into two main areas, both mentioned in the title: the topology part (part I of the book) and the “modern analysis” part (parts II and III of the text, which together comprise a wonderful introduction to functional analysis at the undergraduate level). There are also three appendixes discussing special topics.

Part I of the book covers basic point-set topology. It starts with a chapter on sets (including cardinal arithmetic and some brief discussion of the Axiom of Choice and Zorn’s lemma), and then has a chapter on metric spaces, covering the basic definitions, examples and theorems. With the properties of open sets established in this chapter, the author then proceeds to topological spaces, the natural generalization of metric spaces. After this are chapters on compactness, separation and connectedness (in that order), covering the usual material on these topics, and then there is a chapter on approximation, which is essentially devoted to a statement and proof of Stone’s generalization of the Weirstrass approximation theorem.

The “modern analysis” portion of the book (parts II and III) is essentially an introduction to functional analysis, and to this day I know of no better place for an undergraduate to learn the basics of this subject for the first time. Part II consists of four chapters: first, a chapter reviewing basic algebraic structures (groups, rings, vector spaces, algebras), then a chapter each on Banach spaces and Hilbert spaces, and finally, in a return to algebra, a chapter on finite-dimensional spectral theory (of normal operators on an inner product space). Because of the assumption of finite-dimensionality, this final chapter is pure linear algebra, with no topology or analysis involved, and is intended to motivate the contents of part III of the text. Here, Simmons discusses Banach algebras, particularly commutative ones; the discussion eventually leads to commutative $C^*$ algebras and a version of the spectral theorem for normal operators on a Hilbert space.

Measure theory is never mentioned in parts II and III of the text, so some of the more interesting Banach spaces (notably the $L^p$ spaces) do not appear, but even without these examples, the student will learn a good chunk of the basic theory of functional analysis, including the “big” theorems on the subject (Uniform Boundedness, Closed Graph, Hahn-Banach, etc.), and will be nicely situated for perusal of more sophisticated texts on the subject, such as Conway’s *A Course in Abstract Analysis*.

There are three appendices following Part III of the book under review; they discuss, in order, fixed point theorems, the Hahn-Mazurkiewicz theorem, and the Stone representation theorem for Boolean algebras. The extent to which things are actually proved in these appendices varies. In the first, some results (the Brouwer and Schauder fixed point theorems) are stated without proof; however, the Banach contraction mapping principle is proved, as is, as an application of it, the Picard existence-uniqueness theorem for differential equations. This is a very nice discussion,
and my only regret is that it was relegated to an appendix, rather than appearing as a section in the chapter on metric spaces. The second appendix states the Hahn-Mazurkiewicz theorem, and proves the easy direction. The third (and longest) of the appendices introduces Boolean algebras and rings and gives a full proof of the Stone representation theorem (that any Boolean ring is isomorphic to the ring of open-closed subsets of some compact Hausdorff totally disconnected topological space).

Simmons writes very well, and this book is so clear that any reasonably good student should have no trouble reading it (or most of it, anyway, some of Part III may be difficult) on his or her own. (Better still, I think most students would want to read it.) There are good examples and motivational discussion. There is also a good selection of problems, and I don't recall experiencing an inordinate amount of difficulty with them as a student. (Contrast, for example, my undergraduate experiences with Rudin's *Principles of Mathematical Analysis* and Herstein's *Topics in Algebra*, just to mention two books that were routinely used at my college back then that I could never use as a text now.)

I mentioned earlier that I am aware now of flaws in the book that I was unaware of in my youth. The primary (and, frankly, inexcusable) one is that Simmons never mentions the quotient topology at all. A related problem is that the book is relentlessly slanted towards analysis, so that the geometric aspects of the subject are generally ignored. There is not even a picture of a Mobius strip anywhere in the book.

Another problematic feature is Simmons' definition of a "totally disconnected" space as one in which any two points can be separated by a disconnection of the space. He states in a footnote that the quoted phrase is not uniformly defined in the literature. This may have been true in 1963, but a quick glance at the books on my shelf (specifically, the ones by Croom, Gemignani, Gamelin and Greene, *Munkres* and *Singh*) shows that they, at least, all define the term in the same way, and unfortunately their definition (the connected components of the space are singletons) is not equivalent to Simmons'. (As is always the case when I need to find an example of a topological space satisfying one condition but not another, I looked first at *Counterexamples in Topology* by Steen and Seebach; they use the term "totally separated" to mean what Simmons calls "totally disconnected", and give examples of spaces that are totally disconnected but not totally separated.)

Other issues are generally more minor and concern editorial decisions that I would have made differently. I think the chapter on connectedness should precede the one on separation, simply because, to my mind anyway, the subject of connectedness is more basic and essential to a beginning topology course than is, say, the concept of a completely regular space. In fact, I probably would also have placed the chapter on connectedness before the chapter on compactness, as I think the notion of a space "being in one piece" is somewhat easier for students to grasp than is the fairly abstract idea of an open cover containing a finite subcover. I also think that the section on locally compact Hausdorff spaces would have been better placed in the chapter on compactness, rather than the chapter on approximation.

Moreover, in the years since this book's publication, it has become fairly common for undergraduate texts in topology to at least mention the fundamental group as an introduction to algebraic topology. The book by Croom has a nice, manageable chapter on it, as does the topology text by Gemignani; the books by Munkres, Gamelin and Greene, and Singh all discuss it in even more depth. So this is at least one way in which this text is showing its age, and may be a point against it for any instructor who wants to end the course with a look at this topic.

With regard to the second half of the book, I wish Simmons had discussed compact operators, at least on Hilbert spaces. The basic theory of these operators is, I think, reasonably within the ken of an undergraduate who has learned the material on Banach and Hilbert spaces done earlier in the text, and there are nice applications to differential and integral equations and to spectral theory that would have complimented other material in the text.

For the reasons specified above, I no longer believe that this is the best mathematics book ever written, or even the best mathematics book that I own. I don't even believe it's necessarily the best choice of text for an undergraduate topology course. But it is still a book with much to recommend it. It remains a model of clear, elegant exposition, and I can tell you from personal experience, still vividly remembered after almost half a century, that an undergraduate course on either topology or functional analysis that is based on this book can be very enjoyable indeed.

Mark Hunacek (mhunacek@iastate.edu) teaches mathematics at Iowa State University.

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