Fractional calculus in bioengineering, part 1

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Fractional Calculus in Bioengineering, Part 1

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ABSTRACT

Fractional calculus (integral and differential operations of noninteger order) is not often used to model biological systems. Although the basic mathematical ideas were developed long ago by the mathematicians Leibniz (1695), Liouville (1834), Riemann (1892), and others and brought to the attention of the engineering world by Oliver Heaviside in the 1890s, it was not until 1974 that the first book on the topic was published by Oldham and Spanier. Recent monographs and symposia proceedings have highlighted the application of fractional calculus in physics, continuum mechanics, signal processing, and electromagnetics, but with few examples of applications in bioengineering. This is surprising because the methods of fractional calculus, when defined as a Laplace or Fourier convolution product, are suitable for solving many problems in biomedical research. For example, early studies by Cole (1933) and Hodgkin (1946) of the electrical properties of nerve cell membranes and the propagation of electrical signals are well characterized by differential equations of fractional order. The solution involves a generalization of the exponential function to the Mittag-Leffler function, which provides a better fit to the observed cell membrane data. A parallel application of fractional derivatives to viscoelastic materials establishes, in a natural way, hereditary integrals and the power law (Nutting/Scott Blair) stress—strain relationship for modeling biomaterials. In this review, I will introduce the idea of fractional operations by following the original approach of Heaviside, demonstrate the basic operations of fractional calculus on well-behaved functions (step, ramp, pulse, sinusoid) of engineering interest, and give specific examples from electrochemistry, physics, bioengineering, and biophysics. The fractional derivative accurately describes natural phenomena that occur in such common engineering problems as heat transfer, electrode/electrolyte behavior, and sub-threshold nerve propagation. By expanding the range of mathematical operations to include fractional calculus, we can develop new and potentially useful functional relationships for modeling complex biological systems in a direct and rigorous manner.

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