Jordan-Brans-Dicke Theory

This is primarily an historical survey of the origins and evolution of one of a variety of many scalar-tensor (ST) alternatives to the standard Einstein equations of General Relativity. This review will only address itself to what has become known as the Brans-Dicke, or more properly Jordan-Brans-Dicke, theory based on the 1961 publications of Brans and Dicke. These papers substantially duplicate much of the work of Jordan and his group. However, for several reasons, including the stature of Dicke, this theoretical work from the 1960s was seminal in triggering an explosion of interest, both theoretical and experimental, into Einstein's theory and its alternatives. Until that time the early corroboration of Einstein's equations seemed to preclude interest in investigation of alternatives. From the 1960s on however, ST theories provided a remarkable impetus for renewed exploration into gravitational physics. Perhaps this has been their most significant contribution to the field of space-time physics, now especially in cosmology.

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Introduction, Genealogy

The term "Brans-Dicke," apparently first introduced by Dicke in his 1962 paper on transformation of units (Dicke 1962), is widely used to describe a modification of Einstein's original formulation of General Relativity to bring it into conformity with some form of Mach's Principle. In standard Einstein theory the space-time metric tensor, or more precisely geometry, is the sole quantity describing gravity. Dicke suggested the addition of another, scalar, field. This scalar, \(\phi\), like gravity, has all matter as its source, and thus, in some semantic sense could be described as an extension of the gravitational field from purely geometric to geometric plus scalar, thus the term "scalar-tensor." The scalar aspect of \(\phi\) is important since it cannot be "gauged" away as the metric and connection components can be, at least locally. This particular formalism was published by Brans and Dicke in 1961 (Brans and Dicke 1961) and based on Brans' Princeton PhD thesis (Brans 1961) (unpublished). In this thesis and subsequent papers, both Brans and Dicke were careful to point out that Jordan and others had previously studied and published papers on such a theory, thus the term "Jordan-Brans-Dicke" has also appropriately been used to label this formalism. Jordan's book (Jordan 1955) provides a thorough presentation of the work of him and his group to that date, with formalisms almost identical to those in Brans' thesis. In fact, although these two compound names are widely used in connection with ST theories, many other workers also published papers proposing similar ideas before Jordan, Brans or Dicke. Hubert Goenner (Goenner 2012) has provided a thorough study of this history, which concludes with a long list of names of workers who have proposed, or at least considered, the idea of adding a scalar to supplement the 4-metric of standard Einstein formulation of the gravitational field equations. This list in fact includes Einstein himself in concert with Peter Bergmann as summarized in Bergmann's later article (Bergmann 1948). In addition to Goenner's extensive work, Schucking wrote a general interest article in Physics Today (Schucking 1999) recalling his personal interactions during the early exploration of scalar-tensor ideas by Jordan, Pauli and others, including comments on the political atmosphere of the Nazi regime and its impact on the work of German scientists of the time. Much of the early (pre-1960) work was associated with the higher-dimensional attempts to find a field unifying electromagnetism with gravity, starting as early as 1912 by Nordström. Similarly, later (1920s) proposals along these lines were made by Kaluza, 1921 and Klein, 1926, whose names are most widely used in connection with higher dimensional theories. Another name in this connection is Thirry 1948 who explicitly proposed that the \(g_{55}\), the extra dimensional metric component, might be a field variable which could appear in the field equations as being proportional to \(\sqrt{G}\) and also determined by "matter," which is only electromagnetic at this point. See Applequist, Chodos and Freund (Applequist et al. 1987) for an edited collection of these and other papers, up to fairly recent supersymmetry and string theories.

Actually, if a list of names were to be used for people who independently proposed ST modifications of standard Einstein theory, the resulting compound title would be extravagantly unwieldy. To avoid such complication associated with attribution, this class of theories and their generalizations,
Einstein's GR, the governing equation is the dependence of the inertial mass of a particle
on its gravitational mass. Einstein's equation is
\[ m_i = m_g \gamma \tag{1} \]
where \( m_i \) is the inertial mass of a particle, \( m_g \) is its gravitational mass, and \( \gamma \) is the ratio of inertial to gravitational mass of any test particle. This equation is constant, leading directly to one form of the equivalence principle, the Weak Equivalence Principle (WEP). The WEP states that any test particle, whether massive or massless, travels in a gravitational field with the same acceleration. This principle is empirically tested and has continually increased its limit, leading to the rejection of alternative gravitational theories that violate the WEP.

The theory of General Relativity (GR) provides a framework for understanding gravity as a geometric effect of space-time. In GR, the geometry of space-time is determined by the distribution of mass and energy in the universe. The equations of GR are written in terms of a metric tensor, which encodes the geometry of space-time.

Mach's Principle

Mach's Principle is a generic and variously interpreted term, "Mach's Principle," (MP), is the subject of a collection of papers based on a conference on MP in the book by Barbour and Pfister, 1995. From the variety of views expressed in this volume, it is clear that the phrase Mach's Principle has a long and often controversial history.

The general idea can be traced from the writings of Bishop Berkeley through Newton to those of Ernst Mach, around 1883. For an overview of Mach's ideas, see the article by Staley in Physics Today (Staley, 2013). Einstein mentioned the work of Mach in his earliest writings on General Relativity as an important motivating principle for him in his development of his General Relativistic Theory of Gravitation. As discussed in the following, Einstein argued in his book The Meaning of Relativity, reprinted in 2013, that in his theory inertial forces observed locally in an accelerated laboratory may be interpreted as gravitational effects that give rise to matter accelerated relative to the laboratory. The mention of accelerated reference frames in the context of gravity is of course related to one or more Equivalence Principles.

There is a huge literature on the questions related to the fact that the inertial reference frames in which Newton's mechanics is valid are precisely those which observe the "fixed stars" to be unaccelerated. More precisely IRF's are those which observe some average velocity of the mass in the universe to be constant. One expression of this fact is the idealized "Newton's bucket" experiment: the surface of water in a bucket will be flat if and only if the bucket-fixed observer sees the fixed stars to be not rotating. This almost sounds like an astrological connection between the fixed stars (read constellations) and local behavior on the earth. However, skipping over astrology, the idea grew that this should not be regarded as a coincidence, but rather that there should be some physical field providing the causality. This is the roughest sketch of a general form of what has come to be called Mach's Principle.

Dicke's motivations

Dicke's thinking along MP lines led him to suggest that we should adopt the convention that inertial forces are "real" in some sense, and in fact due to a field interaction between the mass distribution in the universe and local test masses. If so, then it is clear that, by definition, inertial mass, \( m_i \), should be determined by some universal force field, presumably related to gravity. However, to avoid the mistake of using coordinate dependent arguments, a new, coordinate independent (thus scalar) field would be needed. Implicit in this is the assumption that the dimensionless ratio \( \frac{m_i}{m_g} \) of inertial to gravitational mass of any test particle should be constant. In an easy to understand approximation, consider a particle in the (approximately constant) gravitational field near the surface of the earth. Then the force on a particle is, by definition of \( \frac{m_i}{m_g} \) \[ F = m_i g \tag{1} \]
while Newton's first law gives \( m_i \dot{\gamma} = m_g \dot{\gamma} \) \( \dot{\gamma} \) is equal to \( \frac{\gamma}{\gamma} \) for all (test) particles if and only if \( \frac{m_i}{m_g} = 1 \).

Principle(s) of Equivalence

The expression \( \frac{m_i}{m_g} \) is constant, leads directly to one form of the equivalence principle, the Weak Equivalence Principle (WEP) which states that any point particles fall with the same acceleration in a gravitational field. Obviously there are many caveats to be attached to this form of the WEP in order for it to be empirically tested, as will be touched upon later. In fact, Newton himself proposed such an experiment. Toward the end of the nineteenth century Eötvös performed a torsion balance experiment showing, within his error bars, the independence of free fall acceleration from mass. Later Dicke and his group (Dicke 1961) pointed out that there was a very large error bar in the published Eötvös results and re-did the Eötvös experiment using updated and very careful measurements, but again affirming the WEP to a higher accuracy (Dicke 1961), at least in the relatively weak gravitational field of the earth.

In his book The Theory of Relativity (Einstein 2013), page 108f., especially equation (118), Einstein claimed that his purely metric theory would lead to a dependence of the inertial mass of a particle on the gravitational effects from the rest of the universe. However, Brans pointed out (Brans 1962a) that this was only an ephemeral coordinate effect. In fact, the motion of any test particle solely under the influence of gravity is a geodesic with the gravitational effects of all particles embedded in the metric and the connection. If \( m_i \) is to be measured by some known external force, then in Einstein's GR, the governing equation is
\[ m_i \ddot{x}^{\alpha} + \Gamma_{\alpha \mu \nu}^{\gamma} \dot{x}^{\mu} \dot{x}^{\nu} = 0 \]
(\(\mu\nu\)\(\mu\nu\)) However, as

\[ F = \frac{m_i}{m_g} \dot{\gamma} \tag{2} \]
is well known, at any point coordinates can be chosen so that the metric is the Minkowskian and the components of the connection are zero. Thus for a test particle, $\langle m, \phi \rangle$ can be measured as the ratio of force to proper acceleration as in special relativity and will thus be independent of the purely metric gravitational field of Einstein's theory. In other words, the effect of the universe mass on $\langle m, \phi \rangle$ can be completely eliminated by a coordinate transformation, or in more modern terms, can be gauged away. In all these discussions, the phrase "test particle" can be interpreted as one so small in mass and size as to preclude back-interaction as well as tidal effects. However, anticipating Nordtvedt's work, discussed below, for a mass with non-trivial gravitational self-binding energy, there may be other consequences for the WEP if $\langle G \rangle$ is not constant.

In order to illustrate how inertial induction might be the result of a gravitational theory Sciama, 1953 described a "toy" theory to illustrate what a strong version of Mach's Principle might look like. By analogy, recall electromagnetism in pre-relativistic setting in which the vector potential, $A$, is induced by current, $\langle \rho_{\mu} \rangle$ while the electric force is proportional to the time derivative of $A$. Thus the acceleration of the charge induces a force. Sciama used this analogy to build his toy model in which the acceleration of the fixed stars (modeled as a thin, distant, shell) induces a gravitational source for inertial forces. Unlike GR, this vector-based induction of inertial forces cannot be geometrically gauged away. Of course, Sciama did not propose this as a realistic theory but rather to illustrate what a MP might suggest in some sense.

So, what more is assumed by Einstein in the development of his theory? Dicke termed this the Strong Equivalence Principle:

**Strong Equivalence Principle (SEP): The only effect of gravity is to change the geometry**

Dicke presented a nice summary of these topics in his compendium of papers (Dicke 1964). In searching for a way to include some form of a MP, Dicke was led to consider the possible existence of a field that cannot be "transformed away," that is, a scalar.

Which scalar could be used was in turn suggested to Dicke by the "Large Numbers Coincidence, (LNC)" involving cosmological scaled numbers as described for example by Dirac (Dirac 1938). In terms of the values known in the 1930s, perhaps up to the 1950s, the fact is that $|\langle G \rangle|$ is the usual gravitational constant, $|\langle M \rangle|$ and $|\langle R \rangle|$ are the total mass and radius of the visible universe and as usual units are chosen in which $c=1$. There are various interpretations for these last two numbers and in fact $|\langle R \rangle|$ can be interpreted as the reciprocal of the Hubble parameter, $\langle 1/H(0) \rangle$. What Dicke and others emphasized was the remarkable coincidence that (4) when expressed in atomic units results in the outlandishly unlikely canceling of large and small numbers, $|\langle G \rangle| \sim 10^{-40} \text{ GeV}^{-1}$, $|\langle M \rangle| \sim 10^{80}$, and $|\langle R \rangle| \sim 10^{40}$. Of course a modern reader might well reinterpret this as the "Flatness" problem of contemporary standard cosmology which eventually led to the concept of inflation involving a universally coupled scalar field, as discussed below. In fact, as mentioned above, replace $|\langle R \rangle|$ by the reciprocal of the Hubble "constant." Let $R$ be the total radius of the universe, $G$ the gravitational constant, $M$ and $R$ are the total mass and radius of the visible universe and as usual units are chosen in which $c=1$.

In one sense, this might be regarded as closing a circle from the 1930s, LNC, to the twenty-first century, inflation dark energy, etc. Dicke was led to what became known as the BD theory by hoping to satisfy the WEP (verified to high accuracy by his Eötvös experiment) but violating the yet to be verified SEP by adding a scalar field to the metric so that the effect of mass is not only in its effect on geometry. Specifically, rewrite (4) as $|\langle G \rangle| \sim 10^{-40} \text{ GeV}^{-1}$, $|\langle M \rangle| \sim 10^{80}$, and $|\langle R \rangle| \sim 10^{40}$.

In concluding this brief look at equivalence principles, we must note that Nordtvedt (Nordtvedt 1968a) and Nordtvedt (1968b) was able to show that the addition of a variable $|\langle G \rangle|$ results in a violation of the WEP, even apart from tidal effects, for composite bodies bound with gravitational energy. In particular, he was able to compute new deviations in the predictions of the JBD theory from those of Einstein theory for the earth-moon orbital system taking into account the different gravitational binding energy of the two objects. The use of a rector planted on the moon provided orbital data for this binary system which led to increasing bounds on $|\langle \omega_{\min} \rangle|$. Furthermore, in his analysis of this system Nordtvedt took the first steps in developing the Parameterized Post Newtonian (PPN) tools for comparing a wide class of ST and other gravitational theories to empirical observations.

**Brans' thesis**

Brans was a graduate student in theoretical physics, with general relativity as his major interest. It was suggested that he approach Dicke, known at that time to be interested in testing the WEP. Shortly thereafter, Brans and Dicke, but generalizations of the theory are certainly to be expected (Damour and Nordtvedt 1993).
Reciprocal gravitational constant, while the undetermined constant \( \omega \) is dimensionless. An alternative formulation uses a dimensionless field, \( \phi \).

Instead of changing the metric, we can consider changing the notation, \( \phi \). In the original formulation, \( R \) term contains \( \phi \) appears on the left hand side of the gravitational equations with ordinary matter on the right hand side as source of both geometry and \( \phi \). In (19) \( \phi \) appears as just another form of matter, providing the source for geometry. In other words, is \( \phi \) matter or a part of a generalized gravitational field? This point is of course important in deciding whether or not non-negative energy conditions must be imposed on "matter," which contains \( \phi \) in (19) but not in (18).

Note the formalism in (18) through (20) agrees with Dicke's 1962 article, with \( \phi \) to \( \lambda \). This form strongly suggests that the field \( \phi \) plays the role of \( [G^{-1}] \), as suggested by the qualitative arguments leading to (5). In this sense, consider (20) for the case of an idealized universe consisting solely of a static thin spherical shell of total mass, \( M \), radius \( R \). Later, we will present a form for an exact solution in the exterior. For qualitative results, consider the flat metric, with constant \( \phi = \phi_0 \) inside the shell and a static spherically symmetric \( \phi(r) \) outside. Clearly we can assume a solution which goes to a flat geometry and \( \phi \to 0 \) as \( r \to \infty \). Using an expected \( \phi \) form outside, and matching for continuity at the shell we get the gravitational equations, \( \sqrt{\abs{g}} = e^{2\theta} \sqrt{|g|} \) consistent with the Large Number Hypothesis of Edдинton, Dirac et al., or the flatness problem in recent cosmology. Of course, to have a gravitational constant satisfying \( f(\infty) \to 0 \), we must assume \( \phi \) carries the units of a massive bosonic field with \( \mathfrak{L} = \Phi_{,\mu} \Phi_{,\nu} g^{\mu\nu} + m^2 \Phi^2 \) (29).

Conformal transformations

Another scalar field naturally arises in the context of (local) conformal changes of the metric, discussed by Dicke (Dicke 1962). Of course, these transformations of Lorentzian geometries indeed leave null \( ( \mathfrak{Phi} \to \mathfrak{Phi} = \mathfrak{Phi}^2 m^2 \Phi^2 ) \) unchanged. Consider, \( \sqrt{\abs{g}} \) to \( \sqrt{\abs{g}} \) within the shell. Straightforward computation leads to \( R = e^{-\phi}(\theta) \) (24). Using (24) and (25), note that the action involving the \( \phi \) term contains \( \phi \) appears on the left hand side of the gravitational equations with ordinary matter on the right hand side as source of both geometry and \( \phi \). In (19) \( \phi \) appears as just another form of matter, providing the source for geometry. In other words, is \( \phi \) matter or a part of a generalized gravitational field? This point is of course important in deciding whether or not non-negative energy conditions must be imposed on "matter," which contains \( \phi \) in (19) but not in (18).

Once interest in a specific generalization of Einstein theory such as that of (16) has been introduced it was natural to extend the ST theory beyond a Lagrangian of the form (15) to some arbitrary function of \( \phi \), \( \phi_{,\mu} \phi_{,\nu} \) such as \( \mathfrak{L} = \mathfrak{mathcall}[\text{L}_s \phi] \) in (13). This has been a widely used tool for exploring current cosmological problems as discussed below. So for a general ST theory we might expect a Lagrangian of the form \( \mathfrak{L} = \Phi_{,\mu} \Phi_{,\nu} g^{\mu\nu} + m^2 \Phi^2 \) (29). In fact, many forms for (22) can be expressed as \( \phi(R) \) form where \( \phi \) is some function of a single variable, \( \phi \) agrees with Dicke's 1962 article, with \( \phi \to \lambda \). This form strongly suggests that the field \( \phi \) plays the role of \( [G^{-1}] \), as suggested by the qualitative arguments leading to (5). In this sense, consider (20) for the case of an idealized universe consisting solely of a static thin spherical shell of total mass, \( M \), radius \( R \). Later, we will present a form for an exact solution in the exterior. For qualitative results, consider the flat metric, with constant \( \phi = \phi_0 \) inside the shell and a static spherically symmetric \( \phi(r) \) outside. Clearly we can assume a solution which goes to a flat geometry and \( \phi \to 0 \) as \( r \to \infty \). Using an expected \( \phi \) form outside, and matching for continuity at the shell we get the gravitational equations, \( \sqrt{\abs{g}} = e^{2\theta} \sqrt{|g|} \) consistent with the Large Number Hypothesis of Edдинton, Dirac et al., or the flatness problem in recent cosmology. Of course, to have a gravitational constant satisfying \( f(\infty) \to 0 \), we must assume \( \phi \) carries the units of a massive bosonic field with \( \mathfrak{L} = \Phi_{,\mu} \Phi_{,\nu} g^{\mu\nu} + m^2 \Phi^2 \) (29).

In general, null geodesics are not conformally invariant, so the path of particles as predicted by equations derived from (28) differ from those from theories using (14) and (15). Furthermore, it should be noted that the curvature scalar, \( \mathfrak{R} \), is an objective observable (at least in principle), independent of choice of coordinates, or "frame." However, in general any non-trivial conformal transformation of the metric will change the value of \( \mathfrak{R} \), and, in this sense, will predict a different value for a physical observable. However, the discussion of whether or not this is a change in the physical theory is still a source of controversy, (Vollick 2004) (Cateno et al. 2007) (Flanagan 2004). In fact, the question of the objective meaning of conformally related formalisms continues to be an active research area.

Of course, in dealing with any theory in its mathematical form, any transformation of variables can be used to make the equations more tractable, or perhaps to change the theory itself. All that is really required is that predicted quantities can be subjected to experimental verification.

Instead of changing the metric, we can consider changing the notation, \( \phi \). In the original formulation, (16), the field \( \phi \) carries the units of a reciprocal gravitational constant, while the undetermined constant \( \phi \) is dimensionless. An alternative formulation uses a dimensionless field, \( \phi \) consisting solely of a static thin spherical shell of total mass, \( M \), radius \( R \). Later, we will present a form for an exact solution in the exterior. For qualitative results, consider the flat metric, with constant \( \phi = \phi_0 \) inside the shell and a static spherically symmetric \( \phi(r) \) outside. Clearly we can assume a solution which goes to a flat geometry and \( \phi \to 0 \) as \( r \to \infty \). Using an expected \( \phi \) form outside, and matching for continuity at the shell we get the gravitational equations, \( \sqrt{\abs{g}} = e^{2\theta} \sqrt{|g|} \) consistent with the Large Number Hypothesis of Edдинton, Dirac et al., or the flatness problem in recent cosmology. Of course, to have a gravitational constant satisfying \( f(\infty) \to 0 \), we must assume \( \phi \) carries the units of a massive bosonic field with \( \mathfrak{L} = \Phi_{,\mu} \Phi_{,\nu} g^{\mu\nu} + m^2 \Phi^2 \) (29).

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The use of isotropic coordinates simplifies the metric, at least in the case \( T_{(\alpha\beta)} = 0 \). Consider then, \( g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \) \( \phi = \phi_0 + \psi \) \( \mathfrak{L}_{\text{JBD'}} = e^\psi (R - \omega \psi_{,\alpha} \psi^{,\alpha}) + 16\pi G_0 \mathfrak{L}_{\text{matter}} \) \( g_{00} = -1 + \frac{2M}{\phi_0 r} \left( 1 + \frac{1}{2\omega + 3} \right) \). From (37) \( g_{00} = -1 + \frac{1}{r} \left( 1 - \frac{2B}{r} \right) \) \( \lambda = \lambda_0 \) \( C = C_0 \) \( \psi = \ln(\phi G_0) \), with an undetermined dimensional constant, \( G_0 \), of fundamentally arbitrary value. This certainly seems to be contrary to the spirit of having a field determine the value of \( G \) from the distribution of matter in the universe. In fact, the restatement of the formalism with total Lagrangian density \( \mathfrak{L}_{\text{mbox{JBD}}}(L_{\text{mbox{matter}}}) = e^\psi (R - \omega \psi_{,\alpha} \psi^{,\alpha}) + 16\pi G_0 \mathfrak{L}_{\text{mbox{matter}}} \) is equivalent to (16).

Exact Spherically Symmetric Static Vacuum Solution in Isotropic Coordinates

The most easily observed is the famous Mercury perihelion rotation. This was the subject of a good bit of controversy specifically \( \text{JBD}(\omega, 0) \), and must be determined by experiment. \( \omega \) would be expected to be determined by mass. So, in keeping with the historical context of this review, note that in their first step Brans and Dicke considered the first order expansion of the metric used in early Einstein theory.

Approximate Evaluation of Constants and Three Standard Tests

Set \( g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \) \( \phi = \phi_0 + \psi \) \( \mathfrak{L}_{\text{JBD'}} = e^\psi (R - \omega \psi_{,\alpha} \psi^{,\alpha}) + 16\pi G_0 \mathfrak{L}_{\text{matter}} \) To this Newtonian approximation, we then get \( g_{00} = -1 + \frac{1}{r} \left( 1 - \frac{2B}{r} \right) \) \( \phi = \phi_0 e^{\alpha_0 C} \left[ \frac{1 - \frac{2B}{r}}{1 + \frac{2B}{r}} \right] ^{\frac{1}{\lambda}} \) \( g_{ij} = \delta_{ij} \left( 1 + \frac{2M}{\phi_0 r} \right) \) \( G_0 = \frac{2\omega + 4}{2\omega + 3} \frac{1}{\phi_0} \). From (37) \( g_{00} = -1 + \frac{1}{r} \left( 1 - \frac{2B}{r} \right) \) \( \psi = \ln(\phi G_0) \), with an undetermined dimensional constant, \( G_0 \) in terms of the observed effective gravitational constant, \( G_0 \), and also in the manner in which the Einstein limit can be approached for large \( \omega \). Clearly we can regard the change from \( \omega = 0 \) to \( \omega = \infty \) as a renormalization resulting if \( \omega = \infty \), but only in this first order approximation. Of course \( \psi \) is the order is the most easily measured in terms of local evaluations of \( \psi \) by laboratory scaled Cavendish type measurements, so this renormalization would not be empirically significant.

- **Einstein limit**: Empirical predictions of JBD theories will approach those of standard Einstein theory as \( \omega = \infty \). The purpose of observations can be regarded as placing limits on \( \omega \) or in more current terms the PPN parameters. The exploration of stronger gravity has led to results standard effectively eliminating JBD as a viable alternative to Einstein theory, except possibly in cosmological limits.

Earliest formulations for JBD tests.

In the historical spirit of this review, first consider the earliest work of Brans and Dicke in the late 1950s and early 1960s (before the work of Nordtvedt and Will) applying their formalism to the data provided by the classic three standard tests. These were originally designed to compare Einstein's new theory of gravitation to Newtonian gravity. The purpose of observations can be regarded as placing limits on \( \omega \) or in more current terms the PPN parameters. The exploration of stronger gravity has led to results standard effectively eliminating JBD as a viable alternative to Einstein theory, except possibly in cosmological limits.
As Einstein's gravitational theory predicts an amount that agrees with observation to within experimental error. Again, this is a true test of Einstein's General Relativistic metric. For the BD approximations the (null geodesic) passing at distance \( \frac{r}{m} \) from mass \( m \) is bent by an angle \( \frac{4 \pi G \rho m}{r} \) (1 - \( \frac{\omega}{\gamma} \)) for the JBD approximate metric.

Experimental period, 60s and 70s

Coincidently the publication of the 1961 BD paper and the many following papers and experimental suggestions by Dicke occurred during the early stage of a remarkable expansion in the development of rockets and other technology by NASA in the US and similar government funded groups throughout the world. Since General Relativistic effects are most accurately measured in strong gravitational fields, solar system observations were precisely what was needed to convincingly distinguish Newtonian from Einsteinian gravitational theories. Now there was even more motivation because of the relatively wide publicity associated with the BD paper. So the serendipity of the huge increase in funding for solar system exploration and observations with the widely read papers on the existence of a class of viable alternatives to Einstein's gravitational field equations led to an explosion of well-funded interest in General Relativity. As discussed above, an undetermined dimensionless parameter \( \psi \) appeared in the BD formulation. In typical correspondence principle expectations, the "new" ST theories approached Einstein's in the limit of large \( \psi \). Hence, the ST formulation becomes effectively equivalent to the simpler Einstein equations in some limit, in this case as \( \psi \to \infty \).

The new and evolving experimental, observational, work was fortunately accompanied by new theoretical studies, most notably by Nordtvedt and Will who provided additional parameters in the now standard Parameterized Post Newtonian (PPN) formalism. These were designed by an expansion of the most general metric beyond the Newtonian approximations used in the discussion above. They can then be used to compare various general relativistic gravitational theories to each other and to observed data. For the purposes of this brief review the PPN parameter of most significance is \( \gamma \). Clearly the Einstein limit, \( \gamma = 1 \) is equivalent to \( \psi = 0 \). In fact, if \( \psi = 1 \) for small \( \psi \) or \( \psi = 2 \) for large \( \psi \), then \( \gamma \) is both bounded and \( \gamma \to 1 \) as \( \psi \to \infty \). This is called the Nordtvedt effect.

In the terminology used above, this could be classified as TEST IV, which instead of comparing Einstein to Newton gravity, compares Einstein to ST, as illustrated by the example of a binary pulsar pair orbiting in their very strong gravitational fields. Since pure Einstein theory allows only quadruple, or higher multipole, emission, while ST theories would allow dipole gravitational radiation for such a pair, the rate of gravitational energy loss due to radiation provides an important tool to distinguish Einstein from other theories. For this reason the study of binary pulsars in...
connection with confirmation and refutation of theories continues well past the 1970s. See for example the 2013 paper of Antoniadis et al. (Antoniadis et al. 2013).

The 1980s on: Current cosmology and rebirth of interest in variants of standard Einstein theory

In addition to continuing study of gravitational radiation from BP's the 1980s began the intensive study of JBD and other ST theories in connection with the increasing information concerning cosmological parameters available from newly developed observational tools such as the Hubble telescope. As a result, many other variations of Einstein theory have been developed. In particular, current cosmological topics such as Inflation, Dark Matter and Dark Energy have been associated with the development of theories modifying standard Einstein theory involving one or more scalar fields either as a new, yet to be observed "matter," or modifications of the Einstein action, such as f(R) theories.

ST viable wormholes?

Actually, Brans found three other classes of solutions, for ranges of \(\omega\le 0\) and for which the expression for \(\lambda^2\) in (35) is negative. Dicke insisted that these values of \(\omega\) could not be even remotely related to a Mach's principle, so they were not widely noticed, but these exact vacuum solutions can be found in Brans thesis, (Brans 1961a), or in (Brans 1962b). In fact, it turns out that the presence of such solutions when the formalism is interpreted so that the \(\phi\) contributions are effectively combined with matter as in (19), have led to speculation that the violation of the weak energy condition by such Brans solutions of types II, III,IV could permit the construction of wormhole type solutions in the ST theories without the presence of exotic matter. For example see Nandi (Nandi 1997).

References


P. Bergmann, (1948), Unified Field Theory with Fifteen Field Variables, Ann. Math. 49.


Brans-Dicke theory: Jordan vs Einstein Frame. Composite wormholes in vacuum Jordan-Brans-Dicke theory. More Physics 2007. Emergent universe in a Jordan-Brans-Dicke theory. DOI: 10.1088/1475-7516/2007/11/030. Sergio del Campo, Ramon Herrera, Pedro Labrana. In this paper we study emergent universe model in the context of a self interacting Jordan-Brans-Dicke theory. The model presents a stable past eternal static solution which eventually enters a phase where the stability of this solution is broken leading to an inflationary period. We also establish constraints for the different parameters appearing in our model. The action which describes Brans-Dicke theory is given by, 

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{|g|} \left( -\Phi R + \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi \right).$$

In "The Scalar-Tensor Theory of Gravitation", of Yasunori Fujii and Kei-ichi Maeda you can find explicitly the solution, in Appendix C (pag. 195). Personally, I really didn't like this book and even this demonstration it's very difficult to follow. So I did it in another way. Use the usual theory for the GR part, and isolate this term: 

$$\int d^4 x \sqrt{-g} \Phi \delta R_{\mu\nu} \Phi^\mu \Phi^\nu.$$