An important part of nonabelian algebraic topology is concerned with the properties and applications of homotopy groupoids and filtered spaces.

Noncommutative double groupoids and double algebroids are only the first examples of such higher dimensional structures that are nonabelian. Towards noncommutative algebraic topology. Nonabelian KP hierarchy with Moyal algebraic coefficients. Algebraic topology and the quantization of fluctuating currents.


The main theme of this book is that the use of filtered spaces rather than just topological spaces allows the development of basic algebraic topology in terms of higher homotopy groupoids; these algebraic structures better reflect the geometry of subdivision and composition than those commonly in use. Exploration of these uses of higher dimensional versions of groupoids has been largely the work of the first two authors since the mid 1960s.

The structure of the book is intended to make it useful to a wide class of students and researchers for learning and evaluating these methods, primarily in algebraic topology but also in higher category theory and its applications in analogous areas of mathematics, physics and computer science. Part I explains the intuitions and theory in dimensions 1 and 2, with many figures and diagrams, and a detailed account of the theory of crossed modules. Part II develops the applications of crossed complexes. The engine driving these applications is the work of Part III on cubical $\omega$-groupoids, their relations to crossed complexes, and their homotopically defined examples for filtered spaces. Part III also includes a chapter suggesting further directions and problems, and three appendices give accounts of some relevant aspects of category theory. Endnotes for each chapter give further history and references.

Keywords: Algebraic topology, homotopy theory, nonabelian methods, van Kampen theorem, groupoids, cubical homotopy groupoids, crossed complexes, filtered spaces, crossed modules, double groupoids, cubical sets with connections, monoidal closed categories, higher category theory, homotopy classification of maps, classifying spaces