TEACHING NUMERACY: WILL WE EVER LEARN?

Mike Askew

At the launch of the National Numeracy Project, Chris Woodhead placed great emphasis on the need to put 'instruction' back on the teaching agenda. Asked what theory of learning this view was based upon, he replied that he did not have a theory or learning, only a theory of teaching.

I suggest that any theory of teaching has to be based on a theory of learning, however implicit. I therefore want to begin by exploring views on learning and then link these to implications for teaching.

Notions of numeracy, or you're never alone with a fraction

What is this beast called numeracy? Crowther's original concept of 'scientific literacy' seems a long way from current uses of the term. I do not intend to produce a definition of numeracy as I believe it is only through their use that terms come to be tacitly understood. Rather I want to suggest two strong metaphors for learning that might influence how we interpret what it is to be numerate: learning as acquisition and learning as participation.

Learning as acquisition

The first commonly adopted metaphor for learning is the acquisition metaphor. This is deeply embedded in our normative language for talking about learning. Being numerate is thought of as an adjective to describe pupils' 'states': internal cognitions or dispositions. The curriculum is defined as a series of mathematical 'objects' to be passed on to pupils or a set of outcome states pupils need to attain.

One possible consequence of this acquisition view of learning is a separation of the locus of responsibility for learning from teaching. Failure is the responsibility of the individual, either teacher or pupil. If the teacher, this is because they lack the skills to 'pass on' the knowledge (or worse, wilfully withhold it). In a radio interview last year about a small decline in national test results in mathematics at age 11 one of the Government's ministers repeatedly referred to teachers not 'giving' pupils the skills they needed and the Government's commitment to making sure that teachers would 'give' pupils the skills in future.

Alternatively, if the 'fault' does not lie with teachers then it must be that pupils are not capable of 'grasping' ideas. Research into teachers' beliefs about pupils' learning [1] suggests the need for pupils to be 'ready' to learn ideas or that the mind is like a 'leaky vessel' [2] with pupils lacking the ability to 'retain' (ie, hold onto) what has been taught.

Within the acquisition metaphor of learning, a dominant view is that the social acts as the means through which individuals acquire or construct concepts or understandings. Primacy in learning is given to the individual, with the social acting as the 'container' for learning, shaping but not constituting the learning. The individual has a disembodied mind that acquires (or constructs) knowledge.

Learning as participation

In contrast with this view, many theorists are turning to a view of 'learning as participation' and developing theories that attend to the socio-cultural contexts for learning [3, 4 and 5]. Theories of learning as participation do not accept the distinction between the learner and the context within which learning takes place. Learning is not regarded as being put into practice nor is social practice seen as simply the mechanism through which learning is brought about.

Within this paradigm, classrooms comprise 'communities of practice' [4]. Learning within a particular classroom means learning to become a 'full participant in a sociocultural practice'.

While some writers argue for the need for a paradigm shift away from (or even rejecting) acquisition perspectives in favour of participation, I agree with Sfard [6] that the metaphors are not alternatives: both are necessary. Each provides different insights into the nature of teaching and learning.

However, I do suggest that participation in some sense precedes acquisition. Pupils' learning can be examined through analysis of their 'participation in sociocultural activities' [7] and learning is regarded as occurring through changes in such participation. Thus the focus of attention within the classroom is...
the nature and content of the sociocultural activities as determined by the provisions made by the teacher, the interactions of teacher and pupils within the lessons, and the prior understandings of the participants. Analysing classroom practices within a framework of 'transformation of participation' may provide a discourse that examines the sociocultural activities in which pupils have the opportunity to participate. For example, consider the representation in diagram 1.

![Diagram](image1)

Ask teachers or primary school pupils what fraction is represented and most will say 3/5 (or 2/5, or possibly both). However, it is possible to 'read' the diagram in many other ways: it might represent 1/2, 2/3, 1/5 or 1/3. I suggest the reason that it is almost universally read as 3/5 is not to do with the diagram per se, nor with pupils' ability to perceive the fraction within the diagram. Three fifths is the common reading because of well established common practice: everyone from text book writers to teachers to parents 'reads' the diagram as three fifths. A social practice is at the heart of reading the diagram.

A view of social practice as being at the heart of mathematics may help raise awareness of why some pupils experience difficulties with mathematics. A test for 10-year-olds had a question about ships moving around a coordinate grid. There are many facets of knowledge about ships that pupils have to suppress in order to engage with the task as a mathematical one (in the real world ships do not travel along grid lines, they do not turn through right angles, and so forth). In being asked “what must you say to the ship to move it from B to C” pupils have to engage in the practice of treating this 'ship' as sentient. So is it unreasonable for a pupil to answer, as one I came across did, “ready, steady, go”?

In such circumstances, who is to ‘blame’ for the pupil's inappropriate answer: the test writer, the teacher, the pupil? Locating the ‘blame’ for failure within any particular agent is not possible: each displays some lack of awareness of the conventions and social practices within which such test items are located, so responsibility has to be distributed across all participants (including those physically absent).

**Talking about teaching, or show you’re working**

If we accept the participation metaphor for learning, then what are the implications for teaching? In order to explore these I want to make the distinction between pedagogy and didactics.

**Pedagogy**, as I understand it, is concerned with general organisation of pupils for learning. This may not be necessarily linked with a particular discipline and which addresses issues such as:
- lesson structure
- within-class grouping
- between-class grouping

The introduction of the ‘daily dedicated mathematics lesson’ and the suggested three-part structure for lessons are pedagogical imperatives. Through these daily lessons it is intended that all pupils get access to key ideas in mathematics, a sentiment I wholeheartedly endorse. However, we need to attend to the quality of lessons as well as the quantity and style of lessons if learning of mathematics is to improve. In other words, the didactics of mathematics teaching needs to be attended to.

Before turning to aspects of didactics, I want to address one or two pedagogical issues about the grouping of pupils for mathematics lessons. Dylan Wiliam and I [8] have looked at extensive research on the most effective models of in-class grouping. The general message from the evidence is that the benefits of small-group work appear to be most marked when there is an appreciable range, but not the full range of abilities in a group. There is evidence that the extent to which a pupil gives detailed explanations is a good predictor of how much that pupil will gain from small-group working. In groups with a narrow range of attainment, pupils appear to be discouraged from either asking for, or giving, help. On the other hand, if the range of attainment is too great, the middle attainers miss out, as they neither seek nor give help.

With regard to between-class groupings, a number of primary schools appear to be adopting setting for mathematics as a means of meeting the needs of pupils at different levels of attainment. The work of the American researchers Brophy and Good has been much drawn upon in the development of the National Numeracy Strategy. In their most recent review of research they summarise clearly the evidence on between-class ability grouping:

> It is a sensible idea in theory because reducing heterogeneity should make it easier for teachers to meet more of their students' needs more consistently. In practice, however, it is less appealing. Its effects on achievement are weak and mixed rather than reliably positive, and it appears to have undesirable affective and social effects that conflict with the nation's egalitarian traditions [9, page 263].

While the pedagogy of mathematics lessons is one element in making sure pupils gain access to the curriculum, to maximise pupils' opportunities to participate in mathematics, we also need to look closely at the **didactics** of mathematics. There are four key elements that teachers need to attend to. These elements include aspects of the:
- tasks that pupils work on
- talk that takes place during mathematics lessons
tools available to pupils to help them tackle the mathematics
relationships and classroom norms that support learning mathematics.

Mathematical tasks that seem to develop pupils' understandings are ones that challenge them to solve problems rather than work through routine exercises. For example, one teacher had a bottomless triangular pyramid that became a container for marbles when held apex down. Filling this layer by layer, the pupils were challenged to find a way of representing the number of marbles in each layer and predicting the total number of marbles for a given number of layers. This led to rich discussion about number patterns and plenty of mental calculation. Tasks that do not 'set ceilings' on the level of difficulty enable pupils to engage with the mathematics at a number of different levels of attainment.

Talk: where both teacher and pupils feel that they have something to learn from each other creates a rich learning environment. We know that pupils do construct understandings that may not be mathematically correct. Getting them to talk about their reasons and methods helps bring these understandings into the open. Feedback from the teacher and other pupils helps clear up misunderstandings, although this may only happen over a period of time.

Feedback needs to be provided for correct answers as well as incorrect ones, particularly when some explanations may be more powerful than others. One pupil may explain that you multiply by ten by adding a nought and another explain that you are moving the digits up a column. At one level both explanations are correct, but the former ceases to be useful in the context of decimals. As the pupil is not going to know that, the teacher's role is central in exploring the differences in the explanations.

All mathematics uses tools of one sort or another, even if these are simply symbols and diagrams. Mathematics lessons also have tools that not only help pupils do the tasks, but also provide models or embodiments of mathematical ideas. Different pupils have different learning styles, some preferring the visual, some the oral and some the kinaesthetic. Tools for tackling mathematics need to provide a range of experiences to meet different learning styles. This may be why the empty number line seems such a powerful tool: it is visual, you can use it to explain your methods and there is the kinaesthetic element of making the jumps along it.

Turning to relationships and norms, if pupils are going to be challenged with mathematical problems, if they are going to offer explanations and if they are going to feel free to use tools that suit them individually, then the classroom needs to provide a supportive environment where these things can happen. Pupils are very good at intuitively picking up the norms of the classroom and knowing what the hidden rules of the game are. For example, if they are never asked "how did you work that out?" when the answer is correct, pupils suss out that this question really means "you've got it wrong". If norms are going to be changed the changes and reasons need to be made explicit to pupils. In one classroom I visited the teacher was concerned that, although the pupils had begun to get quite good at explaining their methods, the rest of the class did not pay that much attention, being more interested in their own method than anyone else's. The teacher worked on changing this norm by getting pupils to re-explain the method they had just heard before explaining their own method and talking to pupils about her reasons for doing this.

Challenging tasks, teacher and pupils learning from each other, a variety of accessible tools and a supportive atmosphere – mathematics lessons that demonstrate these qualities may go beyond simply following the three-part structure. Classrooms may start to become 'communities of learners'. Communities because all need to work together, taking responsibility for each other's learning as well as their own; and learners because all, the teacher included, need to see themselves as learners.

References

Mike Askew works at King's College London, School of Education. This article is adapted from his opening address at this year's joint ATM/MMA Easter conference.
The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk.

Legitimate uses of this document include printing of one copy for personal use, reasonable duplication for academic and educational purposes. It may not be used for any other purpose in any way that may be deleterious to the work, aims, principles or ends of ATM.

Neither the original electronic or digital version nor this paper version, no matter by whom or in what form it is reproduced, may be re-published, transmitted electronically or digitally, projected or otherwise used outside the above standard copyright permissions. The electronic or digital version may not be uploaded to a website or other server. In addition to the evident watermark the files are digitally watermarked such that they can be found on the Internet wherever they may be posted.

Any copies of this document MUST be accompanied by a copy of this page in its entirety.

If you want to reproduce this document beyond the restricted permissions here, then application MUST be made for EXPRESS permission to copyright@atm.org.uk

**Membership of the ATM will help you through**

- Six issues per year of a professional journal, which focus on the learning and teaching of maths. Ideas for the classroom, personal experiences and shared thoughts about developing learners’ understanding.
- Professional development courses tailored to your needs. Agree the content with us and we do the rest.
- Easter conference, which brings together teachers interested in learning and teaching mathematics, with excellent speakers and workshops and seminars led by experienced facilitators.
- Regular e-newsletters keeping you up to date with developments in the learning and teaching of mathematics.
- Generous discounts on a wide range of publications and software.
- A network of mathematics educators around the United Kingdom to share good practice or ask advice.
- Active campaigning. The ATM campaigns at all levels towards: encouraging increased understanding and enjoyment of mathematics; encouraging increased understanding of how people learn mathematics; encouraging the sharing and evaluation of teaching and learning strategies and practices; promoting the exploration of new ideas and possibilities and initiating and contributing to discussion of and developments in mathematics education at all levels.
- Representation on national bodies helping to formulate policy in mathematics education.
- Software demonstrations by arrangement.

**Personal members get the following additional benefits:**

- Access to a members only part of the popular ATM website giving you access to sample materials and up to date information.
- Advice on resources, curriculum development and current research relating to mathematics education.
- Optional membership of a working group being inspired by working with other colleagues on a specific project.
- Special rates at the annual conference
- Information about current legislation relating to your job.
- Tax deductible personal subscription, making it even better value

**Additional benefits**

The ATM is constantly looking to improve the benefits for members. Please visit www.atm.org.uk regularly for new details.

**LINK:** [www.atm.org.uk/join/index.html](http://www.atm.org.uk/join/index.html)
Teaching requires a professional model, like we have in medicine, law, engineering, accounting, architecture and many other fields. In these professions, consistency of quality is created less by holding individual practitioners accountable and more by building a body of knowledge, carefully training people in that knowledge, requiring them to show expertise before they become licensed, and then using their professions’ standards to guide their work. Teaching, on the whole, lacks this specialized knowledge base; teachers teach based mostly on what they have picked up from experience and from their colleagues.